MATH 135 — QUIZ 11 SOLUTIONS — JAMES HOLLAND
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Question 1. Using the right endpoints, estimate $\int_0^2 6x + 2 \, \mathrm{d}x$ using a Riemann sum $\sum_{k=1}^{n} f(a + k\Delta x)\Delta x$ for $n = 4$.

Solution .:.
Here, we're dealing with the interval $[a, b] = [0, 2]$ so that $a = 0$ and $\Delta x = \frac{2-0}{n} = \frac{1}{2}$. f is the function we're integrating: $f(x) = 6x + 2$. Therefore,

$$\begin{aligned}\sum_{k=1}^{n} f(a + k\Delta x)\Delta x &= \sum_{k=1}^{4} \left(6\left(0 + \frac{k}{2}\right) + 2\right)\frac{1}{2} \\ &= \sum_{k=1}^{4} (3k + 2)\frac{1}{2} \\ &= (3 + 2) \cdot \frac{1}{2} + (6 + 2) \cdot \frac{1}{2} + (9 + 2) \cdot \frac{1}{2} + (12 + 2) \cdot \frac{1}{2} \\ &= \frac{5}{2} + \frac{8}{2} + \frac{11}{2} + \frac{14}{2} = \frac{38}{2} = 19.\end{aligned}$$

Question 2. Calculate $\int \frac{2}{x} \, \mathrm{d}x$.

Solution .:.
Since $\frac{\mathrm{d}}{\mathrm{d}x} \ln|x| = \frac{1}{x}$, it follows that $\int \frac{2}{x} \, \mathrm{d}x = 2\ln|x| + C$. Using the (reverse of the) power rule doesn't work, as $\frac{x^{n+1}}{n+1}$ for $n = -1$ requires dividing by 0.

Question 3. Calculate the following definite integrals: i. $\int_0^{\pi} \sin x \, \mathrm{d}x$ ii. $\int_1^2 -3 \, \mathrm{d}x$.

Solution .:.
i. Since $\int \sin x \, \mathrm{d}x = -\cos x + c$, we get that $\int_0^{\pi} \sin x \, \mathrm{d}x = (-\cos(\pi)) - (-\cos(0)) = 1 - (-1) = 2$.
ii. Since $\int -3 \, \mathrm{d}x = -3x + c$, we get that $\int_1^2 -3 \, \mathrm{d}x = (-3 \cdot 2) - (-3 \cdot 1) = -6 + 3 = -3$.

Question 4. Calculate $\frac{\mathrm{d}}{\mathrm{d}t} \int_3^t e^{x^2} \, \mathrm{d}x$.

Solution .:.
Just by the fundamental theorem of calculus, the function $F(t) = \int_3^t e^{x^2} \, \mathrm{d}x$ has $F'(t) = e^{t^2}$.

Question 5. Calculate $\sum_{n=3}^{20} (n + 1)$.

Solution .:.
Note that

$$\begin{aligned}\sum_{n=3}^{20}(n + 1) = \sum_{n=4}^{21} n = \quad & 4 + 5 + 6 + 7 + \cdots + 20 + 21 \\ &= (1 + 2 + 3 + 4 + 5 + 6 + 7 + \cdots + 20 + 21) - (1 + 2 + 3) \\ &= \frac{21 \cdot 22}{2} - 6 = 21 \cdot 11 - 6 = 225.\end{aligned}$$

Alternatively,

$$\begin{aligned}
\sum_{n=3}^{20}(n+1) &= \left(\sum_{n=3}^{20} n\right) + \left(\sum_{n=3}^{20} 1\right) \\
&= (\qquad 3+4+5+\cdots+20) \qquad + (18) \\
&= \big[(1+2+3+4+5+\cdots+20) - (1+2)\big] + 18 \\
&= \frac{20\cdot 21}{2} - 3 + 18 = 210 + 15 = 225.
\end{aligned}$$